

Math with Lenses – Worksheet

1. A glowing object that is 2.5 cm tall is placed 20.0 cm in front of a converging lens. If the focal length of the lens is 7.5 cm, determine:

a) The distance between the image and the lens.

$$\begin{aligned}
 h_o &= 2.5 \text{ cm} \\
 d_o &= 20.0 \text{ cm} \\
 f &= 7.5 \text{ cm} \\
 d_i &=?
 \end{aligned}
 \quad
 \begin{aligned}
 \frac{1}{f} &= \frac{1}{d_o} + \frac{1}{d_i} \\
 \frac{1}{7.5 \text{ cm}} - \frac{1}{20.0 \text{ cm}} &= \frac{1}{d_i} \\
 \left(\frac{1}{12 \text{ cm}}\right)^{-1} &= \left(\frac{1}{d_i}\right)^{-1}
 \end{aligned}
 \quad
 \boxed{12 \text{ cm} \approx d_i}$$

b) The size of the image.

$$\begin{aligned}
 h_i &=? \\
 \frac{h_i}{h_o} &= \frac{-d_i}{d_o} \\
 h_i &= \frac{-d_i h_o}{d_o}
 \end{aligned}
 \quad
 \begin{aligned}
 h_i &= -\frac{12 \text{ cm}(2.5 \text{ cm})}{20.0 \text{ cm}} \\
 \boxed{h_i &= -1.5 \text{ cm}}
 \end{aligned}$$

c) Whether the image is real or virtual. How do you know?

Real $\Rightarrow d_i$ is positive and h_i is negative

2. An object is placed 9.00 cm from a diverging lens. The object is 4.00 cm tall and the focal length of the lens is 5.00 cm. Determine:

a) The distance between the image and the lens.

$$\begin{aligned}
 d_o &= 9.00 \text{ cm} \\
 h_o &= 4.00 \text{ cm} \\
 f &= -5.00 \text{ cm} \\
 d_i &=?
 \end{aligned}
 \quad
 \begin{aligned}
 \frac{1}{f} &= \frac{1}{d_o} + \frac{1}{d_i} \\
 \frac{1}{f} - \frac{1}{d_o} &= \frac{1}{d_i} \\
 -\frac{1}{5.00 \text{ cm}} - \frac{1}{9.00 \text{ cm}} &= \frac{1}{d_i}
 \end{aligned}
 \quad
 \begin{aligned}
 \left(\frac{-14}{45 \text{ cm}}\right)^{-1} &= \left(\frac{1}{d_i}\right)^{-1} \\
 -\frac{45 \text{ cm}}{14} &= d_i \\
 \boxed{-0.311 \text{ cm} \approx d_i}
 \end{aligned}
 \quad
 \boxed{d_i \approx -3.21 \text{ cm}}$$

b) The size of the image.

$$\begin{aligned}
 h_i &=? \\
 \frac{h_i}{h_o} &= \frac{-d_i}{d_o} \\
 h_i &= \frac{-d_i h_o}{d_o}
 \end{aligned}
 \quad
 \begin{aligned}
 h_i &= -\frac{(-3.21 \text{ cm})(4.00 \text{ cm})}{9.00 \text{ cm}} \\
 \boxed{h_i \approx 1.43 \text{ cm}}
 \end{aligned}$$

c) Whether the image is real or virtual. How do you know?

Virtual \Rightarrow Diverging lens, d_i is negative, h_i is positive

3. A 6.00 cm tall candle is placed 9.00 cm from a converging lens with a focal length of 8.00 cm. Determine the location of the image and the magnification. Is the image upright or inverted?

$$\begin{aligned}
 h_o &= 6.00 \text{ cm} & \frac{1}{f} &= \frac{1}{d_o} + \frac{1}{d_i} & \boxed{d_i = 72.0 \text{ cm}} \\
 d_o &= 9.00 \text{ cm} & \frac{1}{f} - \frac{1}{d_o} &= \frac{1}{d_i} & M = -\frac{d_i}{d_o} \\
 f &= 8.00 \text{ cm} & \frac{1}{8.00 \text{ cm}} - \frac{1}{9.00 \text{ cm}} &= \frac{1}{d_i} & M = -\frac{72.0 \text{ cm}}{9.00 \text{ cm}} \\
 d_i &=? & \left(\frac{1}{72.0 \text{ cm}}\right)^{-1} &= \left(\frac{1}{d_i}\right)^{-1} & \boxed{M = -8.00} \\
 M &=? & & &
 \end{aligned}$$

Inverted
(M is negative)

4. A diverging lens has a focal length of 4.5 cm. An object 5.0 cm tall is placed 4.5 cm from the lens. Find the size and location of the image.

$$\begin{aligned}
 f &= -4.5 \text{ cm} & \frac{1}{f} &= \frac{1}{d_o} + \frac{1}{d_i} & -\frac{9.0 \text{ cm}}{4} &= d_i & -\frac{d_i h_o}{d_o} &= h_i \\
 h_o &= 5.0 \text{ cm} & \frac{1}{f} - \frac{1}{d_o} &= \frac{1}{d_i} & \boxed{-2.3 \text{ cm} \approx d_i} & & \frac{-(-2.3 \text{ cm})(5.0 \text{ cm})}{4.5 \text{ cm}} &= h_i \\
 d_o &= 4.5 \text{ cm} & \frac{1}{-4.5 \text{ cm}} - \frac{1}{4.5 \text{ cm}} &= \frac{1}{d_i} & -\frac{d_i}{d_o} &= \frac{h_i}{h_o} & \boxed{2.6 \text{ cm} \approx h_i} \\
 d_i &=? & \left(\frac{-4}{9}\right)^{-1} &= \left(\frac{1}{d_i}\right)^{-1} & & & (2.5 \text{ cm if } d_i = -2.25 \text{ cm}) \\
 h_i &=? & & & & &
 \end{aligned}$$

5. The magnification of an image is -2.35x. If the height of the object is 6.35 cm and is 8.50 cm from the lens, what is the focal length? \rightarrow converging lens!

$$\begin{aligned}
 M &= -2.35 \times & M &= -\frac{d_i}{d_o} & \frac{1}{f} &= \frac{1}{d_o} + \frac{1}{d_i} & \boxed{f \approx 5.96 \text{ cm}} \\
 h_o &= 6.35 \text{ cm} & -M d_o &= d_i & \frac{1}{f} &= \frac{1}{8.50 \text{ cm}} + \frac{1}{20.0 \text{ cm}} & \\
 d_o &= 8.50 \text{ cm} & -(2.35)(8.50 \text{ cm}) &= d_i & \frac{1}{f} &= \left(\frac{52}{340 \text{ cm}}\right)^{-1} & \\
 f &=? & 20.0 \text{ cm} \approx d_i & & & & \\
 d_i &=? & & & & &
 \end{aligned}$$

6. A 4.0 cm tall object is placed 2.0 cm from a converging lens. If the image is real and is 8.00 cm tall, what is the focal length of the lens?

$$\begin{aligned}
 h_o &= 4.0 \text{ cm} & \frac{h_i}{h_o} &= \frac{-d_i}{d_o} & d_i &\approx 4.0 \text{ cm} & \boxed{f \approx 1.3 \text{ cm}} \\
 d_o &= 2.0 \text{ cm} & -\frac{h_i d_o}{h_o} &= d_i & \frac{1}{f} &= \frac{1}{d_o} + \frac{1}{d_i} \\
 h_i &= -8.00 \text{ cm} & \frac{-(-8.00 \text{ cm})(2.0 \text{ cm})}{4.0 \text{ cm}} &= d_i & \frac{1}{f} &= \frac{1}{2.0 \text{ cm}} + \frac{1}{4.0 \text{ cm}} \\
 f &=? & & & \left(\frac{1}{f}\right)^{-1} &= \left(\frac{3}{4.0 \text{ cm}}\right)^{-1} \\
 d_i &=? & & & & &
 \end{aligned}$$